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Predicting majority rule: Evaluating the uncovered set and the strong point

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Abstract

This paper compares two solution concepts for majority rule decision-making in multi-dimensional settings: the uncovered set and the strong point. Our goal is to determine which of these solution concepts is the appropriate generalization of the median voter theorem to more complex (and more realistic) multi-dimensional majority-rule settings. By making this comparison, we also

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contribute to the debate about the degree of sophisticated decision-making exhibited by experimental subjects and their real-world counterparts. Using data from eleven previously-published majority rule experiments and analytic techniques drawn from geography, our analysis confirms expectations that the uncovered set provides accurate predictions of majority-rule decision-making; and, moreover, that the strong point provides little added insight, either as a solution concept on its own, or as a predictor of where outcomes lie inside the uncovered set.

Keyword

Majority rule; median voter theorem; modeling; spatial; strong point; uncovered set

Because the daily practice of democracy relies heavily on majority rule, an understanding of how democracy works requires an understanding of the logic and inner-workings of majority rule, both at the theoretical and empirical levels. Scholars have made great progress on the former level. As a result of work building on Black (1948), for example, we understand that majority-rule outcomes, given voters with single-peaked preferences in a unidimensional (1-D) space, are likely to be at the ideal point of the median voter, the so-called median voter theorem (MVT). The MVT is widely used in applied work.

Considerably less progress has been made on applications of multi-dimensional models. . We have come to appreciate, thanks to Schofield (1978) and McKelvey (1976, 1979), that the way majority rule works in multi-dimensional policy spaces is inherently more complicated than in the single-dimension case. The accepted wisdom is that with the exception of some special cases it is difficult to predict the outcomes of majority-rule voting in multi-dimensional policy spaces. This limitation is problematic given the equally accepted wisdom that in many real-world settings, preferences and alternatives encompass at least two policy dimensions – and predictions based on a 1-D model and the MVT have limited explanatory power in such multi-dimensional settings.

In previous work we have attempted to address this gap by developing a technique to estimate uncovered sets (UCS) for two-dimensional (2-D) spatial models.[1] While the predictive power of UCS has long been suspected (Miller, 1980: McKelvey, 1986), a series of papers has verified these expectations in a variety of experimental and real-world settings (for a review, see Bianco et al., 2015). However, these findings do not exclude the possibility that another solution concept might do a better job of predicting majority rule. In particular, Feld, Grofman, and Godfrey (2014) have offered the Strong Point (SP) as an alternative solution concept.

In this paper we consider the predictive power of the SP and the UCS in a series of 2-D spatial models. At stake in this comparison is the question of how to generalize the MVT to multi-dimensional settings. Providing scholars with an answer to this question frees them to predict political outcomes on the basis of more realistic spatial models (i.e., incorporating two policy dimensions rather than one. Our analysis compares the SP and the UCS using data from eleven majority-rule experiments.We show that an intuitively appealing comparison, a probit regression using Pareto points as the unit of analysis, is problematic. Accordingly, our analysis draws on methods from geography, including tests for clustering and for variation in the symmetry of outcomes around the SP. We also look for evidence that might support a combination of the two solution concepts, where the UCS provides broad limits on outcomes, and the SP predicts the distribution of outcomes within the boundaries of the UCS. This hybrid is important given the large size of the UCS in many of the experiments. Our results show that the distribution of outcomes in these experiments is inconsistent with the SP – outcomes do indeed cluster, but not as predicted by the SP. Rather, clustering is related to the distribution of players' preferences and rules that allocate agenda power, factors that are accommodated within the UCS framework. Thus, the UCS appears to be the better theoretic tool for analyzing multi-dimensional majority rule.

1. Predicting majority rule: the uncovered set and the strong point

The fundamental motivation for this analysis lies in the majority rule program, an extended effort to predict outcomes of group decision-making under majority rule. That is, given a set of alternatives, and a set of decision-makers with preferences over those alternatives, what outcomes may ensue given majority rule? Of course, outcomes are shaped by procedures that determine the set of alternatives under consideration. However, in general these constraints are endogenous and themselves subject to majority vote. Knowing what decision-makers want, and assuming their control of procedure, what end result we should expect?

The majority rule program relies on the spatial model of voting (Austen-Smith and Banks, 1999), where decision-makers' preferences and policy alternatives are represented as points in space. The extent to which a particular alternative is attractive for a particular player is a function of the distance between his or her ideal point and the policy option. The usual assumption is that there is a set N of n legislators and that each player $i \in \mathrm{N}$ has Euclidean preferences defined by an ideal point, ρ_i.[2] We say that one alternative, $x \in \mathrm{X}$, *beats* another alternative, $y \in \mathrm{X}$, if x is closer than y to more than half of the ideal points.[3] That is, there is a majority coalition that prefers x to y and can enforce it. As noted earlier, when ideal points and proposals are described as points on a line, and voting occurs with majority rule and an open agenda, the median voter theorem (MVT) predicts outcomes which will match the ideal point of the median voter.[4]

Beyond knowing 'what can happen', predictions about the possible end points of majority-rule provide a basis for addressing a wide range of questions about the decision-making process. In legislatures, knowing 'what can happen' provides a baseline for assessing the impact of behaviors such as agenda-setting, strategic voting, and bargaining. It is an article of faith among scholars that these behaviors are an important influence on legislative outcomes; but without a characterization of the baseline, it is difficult to verify these claims or to attribute a particular outcome to their use.

Consider the debate over party organization in the American Congress: one side argues that the majority party can influence the outcome of legislative proceedings through agenda control. The other side argues that agenda control conveys no power and that the majority party's apparent influence stems from the fact that it has more elected members (thus more votes) than the minority. Thus, suppose we see a policy outcome that favors the majority party. One inference is that party leaders used agenda control to produce this outcome – and that absent their efforts, a different outcome would have emerged. The claim may be true, but it is also possible that the outcome would have occurred if party leaders simply let the legislative process play out without intervening. Adjudicating this matter requires knowing what would have happened under unconstrained majority rule.

This dispute embodies a fundamental question about legislative action: do parties matter? That is, if we are trying to explain why a proposal was enacted, defeated, or never brought up for debate, must we consider agenda-setting efforts of majority party leaders as a potential explanatory variable – or are outcomes fully explained by what individual legislators are willing to vote for, with party leaders having no influence beyond the votes they cast as members of the chamber?

These two theories imply very different predictions about the relationship between preferences and policy outcomes. If agenda control conveys an advantage to the majority party leadership, then which party holds majority status will generally alter outcomes, even if the overall distribution of legislators' preferences in the chamber stays the same. Under this scenario, outcomes will also vary with changes in the preferences of majority-party leaders, changes in leaders' agenda power, and changes in the internal structures of parties and the way they conduct business. If, on the other hand, agenda control is irrelevant –'parties don't matter'– then changes in majority status or agenda power will have little if any effect on legislative outcomes. Rather, outcomes will be sensitive to changes in the legislators' preferences. Thus, any attempt to explain outcomes in the contemporary Congress requires resolution of the debate over the influence of party organizations and party leaders, which in turn requires an understanding of majority rule.

1.1 The uncovered set

Three decades ago, the majority rule program appeared to be at a dead end, as then-current results suggested that the MVT did not generalize to voting given multiple policy dimensions, seemingly making it impossible to predict outcomes in these settings (McKelvey, 1976, 1979; McKelvey and Schofield, 1986, 1987). Riker (1981: 447) summarized it in the starkest terms: 'Politics is *the* dismal science because we have learned from it that there are no equilibria to predict. In the absence of equilibria we cannot know much about the future at all'.

Subsequent work identified the UCS as the set of expected results of majority decision-making given multiple dimensions (Miller, 1980, 2007; McKelvey, 1986).[5] Further analysis showed that if voters consider long-term consequences rather than choosing myopically between alternatives, outcomes will lie in the UCS (Cox, 1987). Furthermore, for any status quo point, there exists a two-step agenda that yields a UCS point as its final outcome (Shepsle and Weingast, 1984). Numerous procedural and behavior assumptions, including strategic voting, sophisticated agenda control, and cooperative coalition formation, were also found to result in uncovered outcomes (McKelvey, 1986). Additional work using a grid-search technique for estimating 2-D uncovered sets (Bianco et al., 2004) has tested the UCS's predictive power using experiments (Bianco et al., 2006, 2008), where a very high percentage of experimental outcomes are located inside the UCS, and real-world data (Bianco and Sened, 2005; Jeong et al., 2008, 2009a, 2009b; Kam et al., 2010) that confirms the UCS's predictive power.

1.2 The strong point

In a spatial model, the strong point (SP) is the point that has the smallest win set, measured in Euclidian terms or, in our analysis, in terms of the number of grid points contained in the win set.[6]Feld et al. (2014) show that for any spatial game, there is a single SP. Moreover, win set size increases with distance from the SP. These authors argue that in a majority-rule spatial voting game, the probability that a particular point is a final outcome is a function of its distance to the SP: 'movement toward points with smaller winsets can be considered as a 'centrifugal' force pulling outcomes toward the strong point' (Feld et al., 2014, 300). Elsewhere they refer to the 'strong gravitational pull' of the strong point as an influence on majority rule voting and outcomes.

From the viewpoint of Feld et al. (2014), the appeal of the SP is that it builds on a well-known, easily described concept, the win set. Moreover, the idea that outcomes should cluster around the SP is consistent with the concept of transaction costs: decision-makers would settle on the SP or a nearby outcome because their relatively small win sets make it difficult to locate another outcome that is majority-preferred. The SP can also be thought of as the least contentious point – the outcome with the fewest majority-preferred competitors. Finally, under the logic of the SP, decision-makers only have to determine which of two proposals they prefer, while the UCS assumes that decision-makers look down amendment trees to determine the long-run consequences of their votes – a more complex task.[7] Taken together, the authors argue that compared to the winset-driven logic of the SP there is 'no comparably good 'story' to explain the predictive success of the uncovered set' (Feld et al., 2014).

Feld et al. (2014) also offer some limited empirical support for the SP.[8] Using the same data as in this paper, they show that the mean outcome (average x, average y) of each of the majority-rule experiments is relatively close to the experiment's SP. Moreover, using two of the experiments (T1 and T2) where there is a complete record of voting from initial proposals to adjournment, they show that points with small winsets are more likely to win votes and be chosen as final outcomes.[9] We will discuss both of these findings in our empirical analysis later in this paper.

To illustrate the differences between the SP and the UCS, Figure 1 provides an example of the two solution concepts for one of the experiments used in our analysis, Treatment 1 (labeled T1) from Bianco et al 2008. There are five players in Figure 1, with ideal points denoted by dots; three in the northwest and two at the

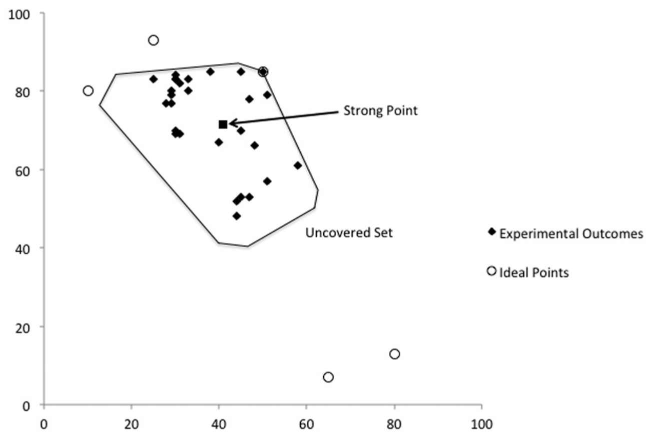

Figure 1. Examples of the Strong Point and the Uncovered Set: The T1 Experiment.

bottom-center region. The irregular shape is the UCS for these five individuals. The SP is the labeled inside the UCS.

Figure 1 highlights that the two solution concepts embody very different expectations about majority rule. The SP provides a singleton location as the expected center of the distribution of outcomes, and posits that outcomes are more likely to be closer to the SP than farther away, meaning that a point outside the UCS could be realized as a final outcome if it is close to the SP. In contrast, the UCS posits that outcomes will always be inside the boundaries of UCS. While in principle all points in the UCS are equally likely to be realized, it is plausible that there will be some clustering due to agenda setting by individuals or groups (Bianco and Sened, 2005; Jeong, 2009a, 2009b).[10] The logic of the SP also implies that the distribution of outcomes should not vary with the size of the UCS. In contrast, if outcomes are constrained by the UCS, then the distribution of outcomes will have a larger support when the UCS is large compared to when the UCS is small.

2. What to do if 'one D is not enough'?

While any model invariably simplifies the behavior it is designed to capture, 1-D spatial models are particularly vulnerable to this charge. The concern is that these models fail to account for the complexity of real-world interactions, that 'one D is not enough' (Aldrich et al., 2007). In many European parliaments, for example, the preferences held by parties or individual legislators reflect beliefs about the size and scope of government, as well as concerns regarding European integration, nationalism, religion, or other factors (Bianco et al., 2014). Similarly, in the modern US Congress, while differences over the size and scope of government separate Democrats and Republicans, other issues – including abortion rights, gun control, and immigration – divide legislators within each caucus, suggesting that a 2-D specification of preferences is needed (Poole and Rosenthal, 2012).

These concerns imply that a 1-D spatial model may not be a good predictor of real-world outcomes – or, even if its predictions are good, this congruence may be largely a coincidence, leaving scholars with a false and overly-simplified impression of the forces driving behavior. For example, in a 1-D model of the legislative process, it is difficult to show how ‘parties matter’ in the contemporary Congress, as primacy of the median voter eliminates the ability of party leaders to shape outcomes by setting the agenda (Krehbiel, 1999; Bianco and Sened, 2005).

However, scholars who use 1-D spatial models have a compelling rejoinder: if they move to two dimensions, how should they predict outcomes? The social choice literature provides two candidates, the SP and the UCS, with no consensus about which is best; and, without such a tool, developing predictions from a 2-D model requires auxiliary assumptions, such as assuming a status quo (Tesbelis, 2002) or restricting the set of possible outcomes (Laver and Shepsle, 1996).

The irony is notable. On the one hand, free software can easily estimate 2-D ideal points using roll calls or survey data. Expert surveys locate party organizations on multiple policy dimensions for a wide range of countries. At the same time, there is a lack of scholarly guidance about how these data should be used.

For the record, the authors of this present paper are firm believers in the efficacy of the UCS. In our view the most important theoretic justification for using the USC is the work cited earlier which shows that uncovered outcomes are the expected result in a wide range of majority rule settings. These findings, developed independently by several scholars using a variety of approaches and underlying assumptions, are not to be taken lightly.[11] Moreover, the UCS’s predictions have been tested in a variety of experimental and empirical settings, including experiments that were conducted before the UCS was conceived. Aside from the simple tests mentioned earlier, there are no corresponding results for the SP.

Even so, there are three motivations for making a serious comparison of the SP and the UCS. First, the creators of the SP (Feld et al., 2014) are accomplished scholars who are equally confident in the their solution concept. Second, it is a fact that some preference configurations yield relatively large uncovered sets. It may be that these large UCSs capture a fundamental uncertainty inherent to majority-rule decision-making – or that another solution concept would explain the within-UCS variation.[12] Third, as Feld et al. (2014) argue, the UCS’s predictive power may be a coincidence, arising because its predictions are correlated with a deeper predictive tool. These concerns are linked to the procedural and behavioral assumptions of the UCS, particularly whether or not decision-makers can predict the long-term consequences of their choices, as the covering relation implies?

3. Testing predictive power: hypotheses and tests

In the eleven experiments we analyze, participants were grouped into five-member committees and given Euclidean preferences over a two-dimensional, 100-point by 100-point policy space.[13] Each participant was assigned a unique ideal point in this space where his/her utility was maximized. Participants were told the location of their ideal points and that their utilities declined as the outcome chosen by the committee moved away from their respective ideal points. By design, ideal points were arranged such that a core did not exist.

Voting in all of the experiments proceeded using an open agenda and a random recognition procedure. The recognized participant would propose a pair of coordinates $\{x, y\} \in \{[0, 100], [0, 100]\}$ to the committee. The proposal could be discussed if participants desired. At the conclusion of the discussion, participants voted openly on the proposal using majority rule. Participants then voted openly on whether to continue voting or adjourn, again by majority rule. If the participants voted to continue, another participant was recognized and the process repeated. If the participants voted to adjourn, the last proposal receiving majority support was the outcome. Participants received monetary rewards proportionate to the proximity between their ideal points and the outcome.[14]

Using these data, our analysis compares two hypotheses. Formally, let p(z) be the probability that a point z with coordinates (x, y) is realized as a final outcome in a real-world or experimental setting where decision-makers choose outcomes using majority rule. Given decision-makers' preferences, let s be the location of the SP with coordinates (i, j), and let d(z, s) be the Euclidian distance between z and s. The predictions of the UCS and SP are as follows:

UCS: $p(z) > 0$ if z is uncovered; $p(z) = 0$ otherwise
SP: $p(z) = f(d(z, s)) > 0$ for all (x, y) and (i, j); with p(z) decreasing in d(z, s)

3.1 A false start: why regression doesn't work

The obvious format for a test between the UCS and SP is a regression setup, where observations for a given voting experiment consist of grid points z contained within the experiment's Pareto set.[15] For each point z, define three variables: (1) whether z is covered or uncovered (*Uncovered*); (2) d(z,s) or the distance between z and the strong point s (*Distance*); and (3) whether z is a final outcome or not (*Outcome*) – and estimate parameters using probit:[16]

$$\Pr(Outcome = 1) = \Phi(b_0 + b_1(Uncovered) + b_2(Distance))$$

Outcome equals 1 if a z is a final outcome and 0 otherwise; *Uncovered* equals 1 if z is uncovered and 0 otherwise, and *Distance* is d(z,s). The UCS hypothesis predicts that b_1 will be positive and significant and b_2 insignificant, while the SP hypothesis predicts that b_1 will be insignificant and b_2 will be negative and significant. The analysis could also show support for both hypotheses with b_1 positive, b_2 negative, both statistically significant.

Unfortunately, this specification raises a series of technical problems. First, the dependent variable has low dispersion. Pareto sets for the experiments typically contain about 4000 points, with between 6 and 40 final outcomes. Even in the best case, the dependent variable will be about 99% 0's and only 1% 1's – a situation that does not make estimation impossible, but does increase the sensitivity of the parameter estimates to stochastic effects. One final outcome in the 'wrong' place (e.g., outside the UCS when the UCS hypothesis is true) can have an outsized impact on the signs and significance of the parameters (King and Zeng, 2001).

Second, it is problematic to estimate parameters for experiments in which the UCS contain all of the final outcomes. The solution is to create some dummy final outcomes that are randomly located in the Pareto Set but outside the UCS, calculate parameter estimates using this augmented dataset, and average parameters across a series of datasets, each with dummy final outcomes in different locations.[17]

Third, the model may have heteroskedastic residuals. To see this, suppose that the UCS hypothesis is true, and final outcomes are randomly distributed inside the UCS. While the probability p(z) that any z is a final outcome is constant within the UCS, the variance in the expected number of final outcomes at a distance d(s, z) from the SP is larger for points with small values of d(s, z) compared to those with higher values – this is because for any two distances $d_1 < d_2$ there are more points at distance d_2 compared to d_1. While this problem can be addressed by using heteroskedastic probit to estimate parameters, it highlights the complexity of a seemingly simple test.

The fourth and most important concern is that UCS and SP variables are highly correlated. This correlation is inevitable given that for most distributions of ideal points, including all of the experiments analyzed here, the SP is near the center of the UCS. As a result, regardless of which hypothesis is true, there will be a strong correlation between d(s, z) and whether z is a final outcome. This correlation is not a problem with probit or any other analytic technique per se – rather, given that the predictions of the two solutions are correlated, it is difficult to estimate their independent effects and obtain accurate inferences about the merits of the two solution concepts.

To illustrate these difficulties, we created a dataset based on the T1 experiment in Bianco et al. (2008), using the actual ideal points and uncovered sets combined with a hypothetical set of 40 final outcomes. The distribution of outcomes is constructed to reflect a situation where the UCS hypothesis is true and the SP hypothesis is false: 30 (75%) of the hypothetical experimental outcomes are randomly distributed inside the UCS, with the remaining 10 (25%) randomly distributed near the UCS as experimental 'close misses'.

Table 1 shows parameters generated using this hypothetical dataset. Working from the right, the estimates show a significant, positive bivariate relationship between the UCS variable and final outcomes – but in the next column, a bivariate, negative relationship between final outcomes and d(s, z). Moreover, in the multivariate estimation, the SP parameter is significant while the UCS parameter is not – even though by construction the SP has nothing to do with the location of final outcomes. The left-most column in Table 1 shows parameters generated using heteroskedastic probit. The model chi-square implies that the residuals are heteroskedastic; a comparison between the two multivariate estimations shows that controlling for this problem reduces the impact of the strong point distance variable by almost 50%.

This analysis also undercuts one of the claims offered in Feld et al. (2014): their finding that points that are close to SP are more likely to be proposed, win one or more pairwise votes, and be realized as final outcomes does not constitute evidence of the SP's explanatory power. As we have shown here, because the SP lies inside

Table 1. Analysis of hypothetical test.

	Dependent variable: point is final outcome			
Independent Variables	Heteroskedastic probit	OLS	OLS	OLS
Distance to Strong Point	–0.033*** (0.01)	0.059*** (0.02)	0.041*** (.013)	–
Outcome In Uncovered Set	–0.31 (0.20)	–0.72 (0.54)	–	0.72** (0.37)
Constant	–1.01 (0.37)	–2.03 (0.72)	–3.2*** (.29)	–4.61*** (0.32)
Model Chi Square	12.72***	14.42**	12.75***	4.2**
N	2457	2457	2457	2457
Chi Sq. Test for Heteroskedasticity	3.41*	–	–	–

*** = significant at 0.01 or better, ** = significant at 0.05 or better, * = significant at 0.10 or better, all two-tail.

the UCS, and because the predictions of these concepts are to some degree correlated, the relationship identified by Feld et al. (2014) would hold even if the SP has no explanatory power at all.

In sum, while a multivariate probit model would seem the obvious choice for testing the UCS and SP hypotheses, in practice this approach is bedeviled with a variety of problems. Accordingly, the next section specifies a test using spatial analysis tools borrowed from Geography.

3.2 Analyzing spatial data: tools from geography

Analyses of spatial data in the discipline of geography often focuses on the existence of clusters – comparing the location of natural objects (e.g., trees or other vegetation) or artificial objects (e.g., buildings), in relation to each other, to some central tendency, or to multiple centers. In our analysis, we implement two canonical tools from geography, and build on these tools to conduct additional tests on our experimental data.

Our first task is to analyze the distribution of outcomes in each experiment for complete spatial randomness (CSR) or the presence or absence of clustering of outcomes (Diggle, 2003). Specifically, the CSR test involves calculating the average distance between experimental outcomes (in the language of geography, finding each outcome's 'nearest neighbor'), and comparing this distance to what would be expected if the outcomes were randomly distributed throughout the space; here, each experiment's UCS.[18] A finding that the distance to the nearest neighbor is sufficiently less than the expected distance given a random distribution implies clustering; a finding that the average distance is sufficiently larger indicates dispersion; values in the middle fail to reject the null of a random distribution.[19]

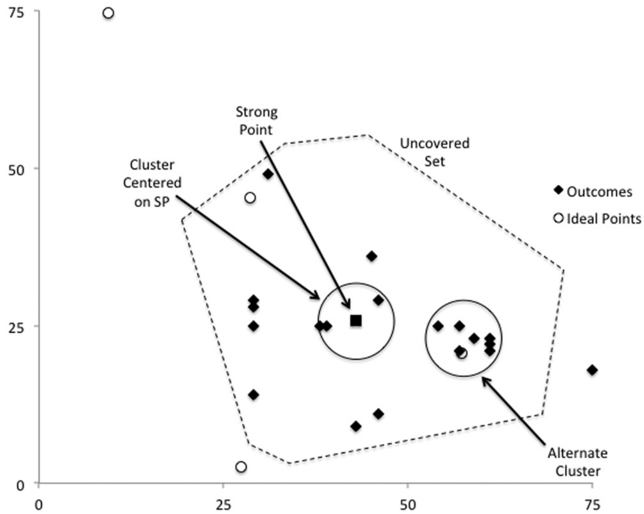

Figure 2. Example of CSR Test and Clustering Analysis: The Two Insiders Experiment.

Recall that the SP predicts that outcomes will be clustered around the SP, with more outcomes close to the SP than farther away. In contrast, the UCS sets firm boundaries on the location of outcomes – the validity of these boundaries have already been confirmed for all of the experiments here (Bianco et al., 2006). However, the logic of the UCS does not specify how outcomes will be distributed inside these limits. Thus, a finding that outcomes in a particular experiment are clustered is a necessary condition for confidence in the SP. However, confidence also requires that the SP be at the center of the largest cluster of outcomes. In contrast, if there is no clustering around the SP, or if the largest cluster is located at some distance from the SP, then it is reasonable to conclude that the SP is not driving the distribution of outcomes.

An example of these conditions is shown in Figure 2, which shows ideal points, outcomes, the strong point, and uncovered sets for Two Insiders experiment (Laing and Olmsted, 1978).

While inspection confirms that outcomes are clustered, the location of the clusters is inconsistent with the SP. Consider the two circles in the figure. The first, labeled 'Cluster Centered on SP', is a 10-unit diameter circle centered on the SP.[20] It contains three outcomes. However, there is another area of the space, 'Alternate Cluster', where a 10-unit circle contains 8 outcomes. Thus, to the extent that there is clustering in the Two Insiders experiment, it occurs in areas that are far away from the SP. This distribution is inconsistent with the SP's predictions – but it is consistent with the UCS, as all outcomes except one are contained within the boundaries of the set.

Figure 2 (together with Figures 1, 4, and 6) also illustrates a fundamental problem with the claim by Feld et al. (2014) that if mean outcomes (average x, average

Table 2. Results of CSR analysis.

Experiment	Distance Between outcomes, expected/actual	Inference (*t*-probability)	N in Cluster, SP/Largest Alt. (*t*-probability)
Two Insiders	1.51	Clustered (.0004)	0.38 (0.13)
King	1.62	Clustered (.003)	3.0 (0.26)
House	1.36	Clustered (.003)	0.33 (0.12)
Wilson	1.27	Clustered (.007)	0.50 (0.11)
PH	1.33	Clustered (.03)	4.0 (0.001)
T1	1.23	Clustered (.03)	0.67 (0.49)
Bear	1.14	Clustered (.12)	0.25 (0.03)
PHR	1.47	Clustered (.24)	2.35 (0.006)
MOW	1.05	Clustered (.33)	0.33 (0.05)
Skew Star	0.81	Dispersed (.02)	–
T2	0.49	Dispersed (<.0001)	–

y) in each experiment are close to the SP, individual outcomes will also be clustered around the SP – and that the SP has predictive power. A look at all of these plots will reveal that there are relatively few outcomes close to the SP, meaning that the closeness of the mean outcome arises only because the individual-level deviations from the SP are being averaged away. More generally, the congruence of mean outcomes with the SP says nothing about whether individual outcomes are clustered around this point.

Table 2 shows the results of the CSR analysis across all 11 experiments. The first column of the table gives the ratio of the expected to actual distance between outcomes – a number greater than one indicates clustering (the experiments are sorted by the size of this ratio), and less than 1 indicates dispersion. For example, in the case of Two Insiders, the ratio is 1.51. The second column interprets the ratio (clustered or dispersed), and gives the t-probability for the likelihood that the null hypothesis (random distribution) is true given the data. As the table shows, all but two of the experiments exhibit clustering, most at levels that are clearly statistically significant.

The last column in Table 2 assesses whether the outcomes in each experiment are clustered around the SP, showing the ratio of the number of outcomes inside a 10-unit circle centered on the SP compared to the largest number of outcomes in any 10-unit circle that does not overlap with the SP circle.[21] For example, Figure 2

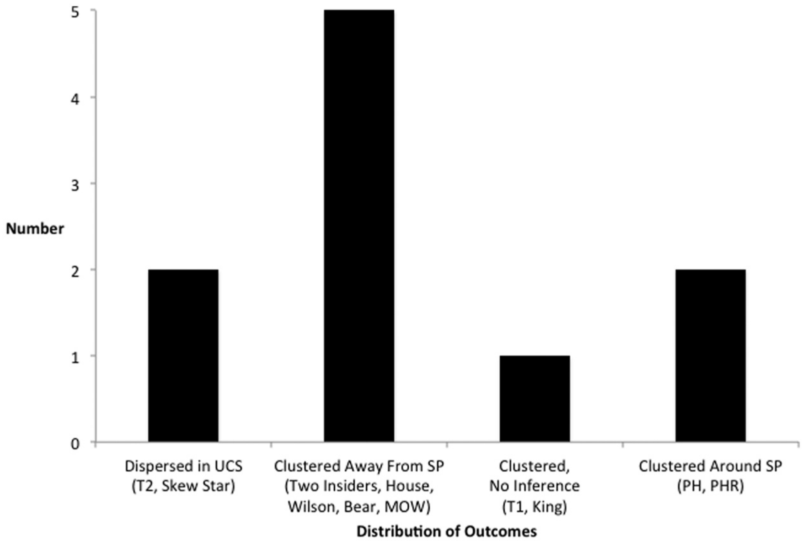

Figure 3. Inferences from CSR Analysis.

shows that in the Two Insiders experiment, the SP cluster has 3 outcomes, and the densest non-overlapping alternate cluster has 8, so the ratio reported in Table 2 is 3/8 = 0.375 = (0.38.) The number in parentheses in Table 2 is the *t*-probability for the difference between the number of outcomes in each circle tested against the null of no difference.

Taken together, the numbers in Table 2 allow us to judge the degree to which the outcomes in each experiment are consistent or inconsistent with the SP, as shown in Figure 3.[22]

Of the 11 experiments, two are consistent with the SP, with outcome distributions that are clustered around the SP. These clusters are, moreover, significantly larger than all of the other clusters in the space. However, two other experiments exhibit no clustering at all; their outcomes are dispersed. In five other experiments, there is clustering, but the largest clusters are nowhere near the SP, and these alternate clusters are significantly larger than the SP cluster.[23] In the final two experiments, there is clustering, but the data are insufficient to make a judgment about the location of the clustering relative to the SP.[24] Overall, these findings lean away from confirming the SP as a solution concept for majority rule games.

3.3 The Quadrat test

Our second test is developed from the Quadrat Method (Shiode, 2008) for discerning the nature of item clustering. This method divides a space (e.g., a forest) into a series of equal-sized areas, then counts the number of items (e.g., trees) in each area, with the goal of finding areas with a disproportionate number of items, implying that these spaces contain clusters or the center of clusters. Our version of this test

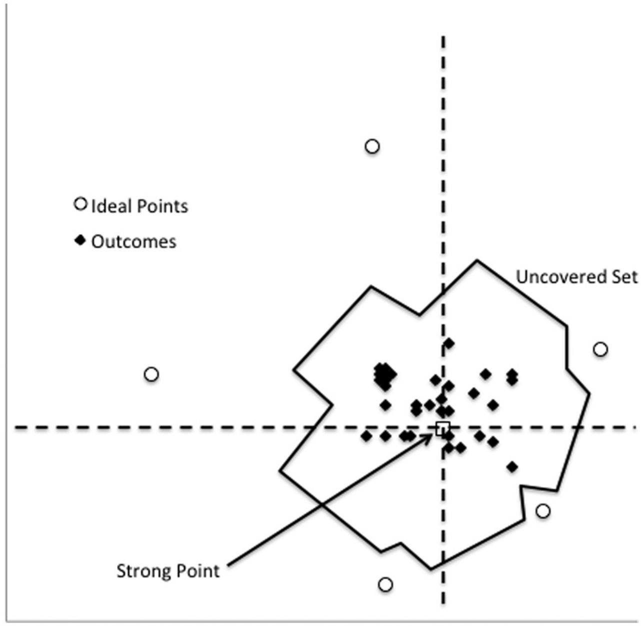

Figure 4. Example of the Quadrat Test: The PHR Experiment.

divides the outcome space for each experiment into quadrants centered on the SP, shown in Figure 4 for the PHR experiment (McKelvey and Ordeshook, 1984).

Our expectation is that if the SP is correct, and that the only factor determining whether a particular point is an outcome is the point's distance to the SP, then outcomes should be distributed evenly (within the limits of random variation) across quadrants. Conversely, if the UCS is the driving force behind outcomes, then for each experiment, the percentage of outcomes in each quadrant should vary with the percentage of the UCS contained in these quadrants. In the case of PHR, the percentage of outcomes varies significantly, from upper-left quadrant to only four in the lower-left.

The PHR experiment also provides a clue about a factor that could influence the location of clusters under majority. Note that the quadrant containing the most outcomes (upper-left) is also the one containing the largest number of players (two versus one in each of the other quadrants). This correlation makes sense given that the players are both the source of proposals and of motions to adjourn. One expects that a player is more likely to offer a motion to adjourn insofar as the status quo is close to their ideal point, that is, in the same quadrant.[25] Having another player or players in the quadrant increases the changes that the adjournment motion will be successful.

To test these hypotheses, we find the percentage of outcomes and UCS contained in each quadrant in eight of our eleven experiments, and estimate the following regression:[26]

$$O_{xy} = b_0 + b_1(UCS_{xy}) + b_2(Players_{xy})$$

Table 3. Results of Quadrat regression.

	Dependent variable: % outcomes in Quadrant x of Experiment y
% UCS in Quadrant x of Experiment y	0.54** (0.26)
N Players in Quadrant x of Experiment y	0.07** (0.03)
Constant	0.02 (0.09)
Log Likelihood	26.32
N	32

Note: *** = $p < 0.05$, two-tail.

where O_{xy} is the percentage of outcomes, UCS_{xy} is the percentage of the UCS, and $Players_{xy}$ is the number of players, all in quadrant x for experiment y. This specification allows a direct test between the UCS and SP's predictions. The SP predicts that b_0 will be significant and near .25, while b_1 will be not significant. The UCS predicts that b_0 will be non-significant, while b_1 will be positive and significant. A finding that b_2 is positive and significant would confirm our agenda-setting expectations, but is consistent with both the UCS and (generously) with the SP.[27]

Table 3 shows the parameters for this regression estimated using the Stata GLM procedure.[28] The parameters indicate that the distribution of the UCS across quadrants shapes the distribution of outcomes across quadrants, as does the number of players, with both variables statistically significant. In contrast, the constant term is insignificant, implying that there is no floor for the percentage of outcomes in a quadrant, as suggested by the SP.

Figure 5 offers an interpretation of these parameters, showing the predicted percentage of outcomes in a quadrant given the amount of the UCS in the quadrant and the number of players.

Mindful of our limited data, we reported predicted proportions as a function of the number of players only for the ranges observed in our data. For example, across the eight experiments and 32 quadrants, the cases where there are two players in a quadrant a range of UCS proportions between approximately 0.20 and 0.35, thus the predicted line for this case is plotted only between these values. The message of these plots is echoes the parameter estimates: rather than observing a uniform distribution of outcomes across quadrants, as predicted by the SP, the distribution is sensitive to the shape, size, and location of the UCS.

4. Implications

Our results support the claim that the UCS is the appropriate generalization of the median voter theorem to 2-D majority-rule settings. As discussed earlier, while the

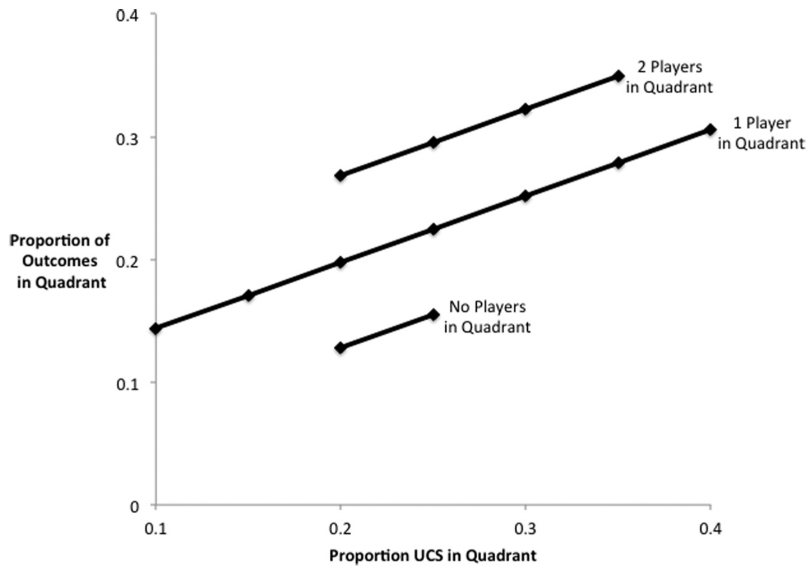

Figure 5. Predicted Distribution of Outcomes Across Quadrants.

UCS has considerable theoretic and empirical support, it has never been tested against an alternate solution concept such as the SP. Our analysis of 11 sets of experimental outcomes finds little support for the SP, either as a predictive method in its own right or as a way to explain the location of outcomes inside the UCS. In particular, we find no evidence that outcomes are clustered around the SP or that they are equally distributed across quadrants. Both predictions are fundamental to the SP's characterization of majority-rule decision-making. Moreover, while there is considerable evidence that the boundaries of the UCS constrain majority rule, there is no evidence that the location of the SP has an independent effect as an influence on the distribution of outcomes within the UCS. Rather, clustering inside the UCS appears to be driven at least in part by the location of players' ideal points.

Another possible source of clustering is suggested by the Wilson (1986) experiment shown in Figure 6, where half of the trials used a backwards agenda, while half used a forward agenda.[29]

The distribution of experimental outcomes confirms Wilson's prediction that backwards agenda trials would produce outcomes near the status quo – note the large cluster near the status quo in the figure. More generally, we expect that clusters in the other experiments reflect variation in the (random) allocation of proposal power across the participants and that in real-world settings, rules that convey agenda control play a crucial role in shaping the location of final outcomes within the UCS.[30] Alternatively, in real-world situations, clustering may arise if the set of potential outcomes is lumpy or otherwise non-continuous, such as in cases where outcomes take the form of committee assignments or the selection of leaders in a legislature.

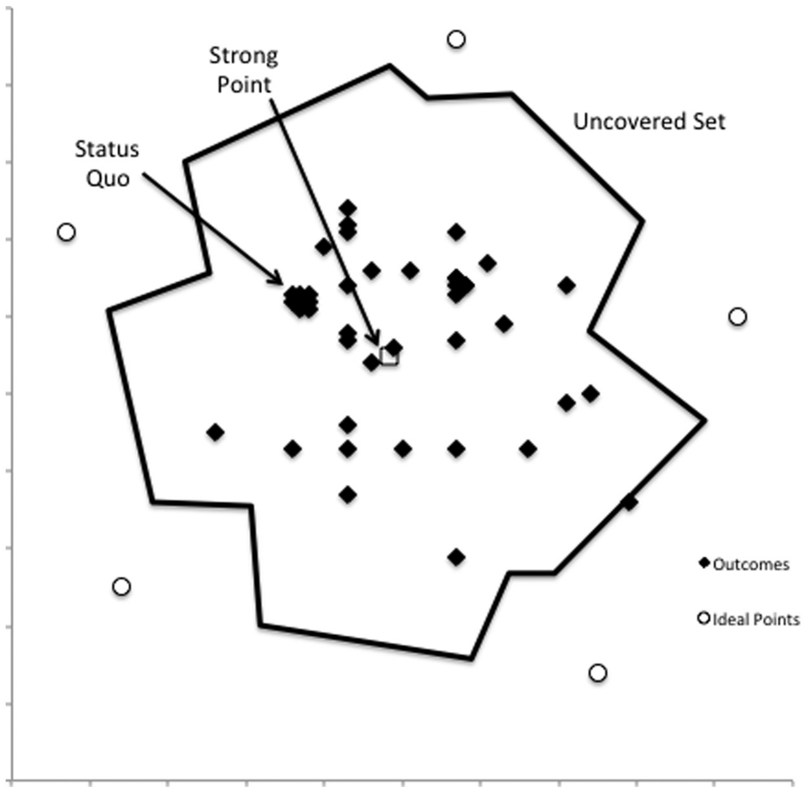

Figure 6. Rules of Procedure and Clustering: The Wilson Experiment.

Finally, our analysis supports a model of decision-making in which individuals are able to make fairly complex calculations, both in assessing the relative merits of different alternatives and in accounting for the consequences of their votes. In particular, one criticism of the uncovered set is that it assumes participants in a majority-rule setting make decisions by determining the long-term consequences of their votes, rather than making pairwise comparisons of the two alternatives being voted on at any particular time. While we have no insights into how the participants in these experiments decided how to vote, our analysis indicates that whatever logic they used is consistent with the UCS's predictions.

5. Conclusion

Our analysis reveals strong support for the uncovered set over the strong point as a predictor of majority rule outcomes in 2-D spatial models. The results also validate the underlying behavioral assumptions of the UCS. Even in abstract experiments, where voters lack cues or context to shape their decisions, outcomes are consistent with the assumption that voters are sophisticated rather than myopic.

The new tools used in our analysis for analyzing clustering and the overall distribution of outcomes in a spatial context are applicable to many other research questions. For example, analyses of the contemporary Congress highlight the importance of intra-party groups such as the Tea Party Caucus for House Republicans and the Blue Dogs for House Democrats in determining party strategies and the content of policy proposals. The question is, to what extent are these groups distinct from the rest of the party – that is, are the respective party caucuses completely spatially random, or are there clusters within each caucus? It may be that other, as-yet unidentified groups in each caucus exert a disproportionate influence over outcomes – if so, the first step in understanding this phenomenon would be to identify the groups and their associated preference clusters, in order to assess their impact over legislative outcomes.

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Notes

1. Of course, spatial models with two dimensions are a special and simpler case of multidimensional spatial models. However, analysis suggests that moving beyond two dimensions does not provide much additional explanatory power.
2. As a matter of norm and convenience, the cardinality of **N**, *n*, is assumed to be the **odd** number of legislators.
3. Lower case *x*, *y* and *z* denote elements of the set of all possible outcomes, a set that is denote by **X**.
4. In a 1-D model, the predictions of the UCS and the SP match those of the MVT.
5. Formally, let x, y, and z be elements of the set X of all possible outcomes. A point x beats another point y by majority rule if it is closer than y to more than half of the ideal points. A point x is covered by y if y beats x and any point that beats y beats x. The uncovered set includes all points that are not covered by other points.
6. For a point x, w(x), the win set of x is the set of all points that are majority-preferred to x.
7. While our focus is on comparing predictions rather than assessing the validity of underlying assumptions, these claims about the theoretic attractiveness of the SP are open to question. While the covering logic which underlies the UCS is not intuitively obvious, the logic of win sets shares this characteristic. In addition, while the SP assumes constant transaction costs, these costs may vary with other factors – some proposals may have high transaction costs because they require particular coalitions for enactment, with no regard for the proposal's distance to the SP. More generally, without a way to determine the magnitude of transaction costs associated with different outcomes, there is no way to determine how these costs enter into decision-makers' calculations.

8. It should be noted that the authors of this paper did not participate in Feld et al's analysis.
9. A third prediction, that experiments where the SP's winset is small will have less variation in outcomes, finds only limited support.
10. For example, when the majority party controls the agenda as in the contemporary House, the expectation is that outcomes will be clustered in the area of the chamber UCS that is closest to the UCS of the majority party.
11. This stream of papers contradicts Feld et al's (2014: 291) assertion that the predictive power of the UCS has only been established for 'king of the hill' voting games.
12. We have argued that exogenous institutional factors, such as party organization, can have this effect. For example, when the majority party controls the legislative agenda, we expect it to choose procedures that yield outcomes in the intersection of the chamber and majority party uncovered sets – or, if no intersection exists, the points in the chamber UCS that are closest to the majority party UCS. The concern mentioned in the text is that similar effects may arise because of more primitive factors, such as the distribution of preferences.
13. The exception to the number of players is the King experiment, which had 7. In cases where the policy space was not 100 x 100, we have rescaled the data for comparability. Appendix 1 provides descriptive data on the experiments, as well as citations to the original publications, which contain more detailed descriptions of the experiments. As a way of familiarizing the reader with these experiments, we use many of them to illustrate our analysis throughout the paper.
14. Participants were informed in advance of the range of payoffs and how their payoff would vary as a function of the final outcome. Potential payoffs ranged from $5.00 to $20.00 per experimental run in then-current dollars. Of course, all of the participants were undergraduate students of (as best we can tell) a wide range of majors. It is difficult to say f results were affected by the subject pool is impossible to say. However, the fact that we have seen consistent results across a wide range of experiments devised by different scholars, plus the corresponding empirical results, suggests there are no obvious biases in the experimental data.
15. In 2-D majority rule experiments such as those presented here, the Pareto set is all points contained within the perimeter around the participants' ideal points.
16. One might well use logit in place of probit.
17. Another solution would be to restrict attention to uncovered points, and estimate a bivariate equation where the only exogenous variable is distance to the SP. Unfortunately, this specification cannot reveal whether both the UCS and the SP have independent effects on outcomes. Second, because most experiments have some final outcomes outside the UCS, focusing on only uncovered points creates a censored sample, with biased parameters the likely result.
18. The test involves calculating d_e the expected distance between outcomes, which equals 1/(2*sqrt(λ), where λ is the density of outcomes, calculated as the number of uncovered points divided by the number of outcomes. The standard error of d_e equals sqrt((4-π)/(m4$\pi\lambda$), where m is the number of outcomes and $\pi = 3.14159$. With these values in hand, (d_m–d_e)/se(d_e), can be evaluated as a z-score. Positive significant values imply dispersion of experimental outcomes; negative significant values imply clustering. Note that using the Pareto set would not allow a clean test between the two solution concepts – if the UCS is in fact driving the location of outcomes, we would expect outcomes to be clustered in the UCS relative to the Pareto set, especially if the UCS is small relative to the Pareto set.

19. Formally, a dispersed distribution is one where the average distance between outcomes is greater than the expected distance given a random distribution (in a random distribution, we would expect actual distance to equal expected distance). It is not hard to construct a dispersed distribution – consider a set of outcomes that are randomly distributed throughout a 2-D space, such that the average distance equals expected. Then, suppose we move some of the points in order to increase the average distance (such moves are always possible). The resulting distribution is dispersed.
20. The diameter of this circle is arbitrary. We have used a variety of differently-sized circles for analyses of clustering, generating results that are essentially identical to those described here.
21. We used a grid-search procedure to search for clusters – that is, for each point in the space (including the SP), we counted the number of outcomes within a 10-unit circle of the point. The ratio reported in Table 2 excludes all clusters where the 10-unit circle overlapped with the circle around the SP – that is, the ratio we report only considers clusters with a different set of outcomes than those that make up the SP cluster. We have varied the size the circle used to define clusters and generated results that are very similar to those reported here.
22. We know that the distribution of outcomes in all of the experiments is consistent with the core prediction of the UCS, that is, that the outcomes will fall inside the boundaries of the UCS
23. Our discussion here uses a .20 probability level to judge statistical significance.
24. This test is designed to be favorable to the SP. If we allowed partial overlaps (for example, the center of the alternate cluster can be on the edge of the SP cluster), for all of the experiments, there is always an alternate cluster that is larger than the cluster around the SP.
25. We expect that the probability of adjournment depends on the distance between a setter's ideal point and the status quo – we cannot test this hypothesis because we do not have the full history of proposals offered in each experiment.
26. We omit two experiments (King and MOW) because they have eight or fewer outcomes, and the T2 experiment because the SP and the UCS are close to the edge of the 2-D outcome space, meaning that the quadrants vary wildly in size.
27. Notwithstanding Feld et al's (2014) prediction that points close to the SP are more likely to be realized compared to those at some remove, they also concede that outcomes may be shaped by factors such as the location of ideal points, the allocation of agenda control, and coalition-building among the players. However, they do not offer any details on what the relationship might look like between these factors and the SP's alleged 'gravitational pull'.
28. We have run the analysis using various weighting schemes and jackknife procedures, and report the baseline GLM parameters because none of the corrections significantly altered the results.
29. In a forward agenda, voting begins with the status quo put against an alternative, with the winner placed against another alternative until a majority votes to adjourn. In a backwards agenda, two proposed alternatives are voted on pair-wise, with the winner placed against another alternative until the last vote, which pits the surviving alternative against the status quo. With the exception of Wilson's experiment, in which half of the trials used a forward agenda and half used a backwards agenda, all of the experiments discussed here used a forward agenda.
30. Unfortunately, the records needed to confirm this hypothesis do not exist for most of the experiments analyzed here.

Appendix 1. Experiments used in analysis.

	Experiment								
	T1	T2	Two Insiders	House	Skew Star	Bear	PHR	PH	Wilson
Source	Bianco et al 2007	Bianco et al 2007	Laing, Olmsted 1978	Laing, Olmsted 1978	Laing, Olmsted 1978	Laing, Olmsted 1978	McKelvey, Ordeshook 1984	McKelvey, Ordeshook 1984	Wilson 1986
N proposals	261	185	NR	NR	NR	NR	NR	NR	641
N Outcomes	28	28	19	19	18	19	33	33	42
N points in Pareto Set	2457	1676	3695	6704	4234	4701	2799	2730	4898
N Points in UCS	1447	226	2623	3744	2531	2488	1505	1482	3615
Location of Strong Point	40.97, 71.61	67.90, 18.55	42.90, 25.85	64.26, 38.80	46.04, 48.27	59.38, 40.88	69.16, 31.23	57.74, 36.37	48.34, 55.93
N Outcomes in UCS	27	17	18	14	15	17	33	32	40

NR = not reported

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